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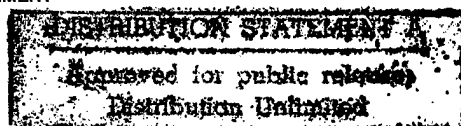
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13. ABSTRACT (Maximum 200 words)

I investigated a problem of extracting a signal from observations corrupted by additive noise, possibly heavy-tailed. More specifically, I assumed that the observation noise is a Levy process, while the signal is Gaussian.

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Summary of Investigation:  
N00014-96-1062

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I investigated a problem of extracting a signal from observations corrupted by additive noise, possibly heavy-tailed. More specifically, I assumed that the observation noise is a Lévy process, while the signal is Gaussian. In addition, I assumed that the observations arrive in discrete times, because any digital signal processing system can only sample at a finite rate limited by the speed of an analog-to-digital converter, and can only process observations at a finite rate determined by the available processor power.

Practical applications in which the observation noise can be represented as a Lévy process are found in engineering problems. For example, the tracking error in code tracking loops, such as those used in Global Positioning System (GPS) receivers. The tracking error (the time offset between the received signal and the internally-generated original signal) usually exhibits heavy tail noise as the tracking slips forward or backward by one time step. Another example is a phase tracking process. When the number of significant interference sources is small and when interferers are pulsed or hopped, the noise and interference is described better by the combination of a Wiener process and a jump process.

In order to extract a Gaussian signal contaminated by a Lévy noise, I constructed a non-linear recursive filter that minimizes the  $L^2$  error. Since implementing this optimal filter requires excessive computation, I proposed a more practical sub-optimal filter that approximates the optimal filter. In fact, the performance of this sub-optimal filter is near by optimal as long as observations arrive frequently. I tested the performance of this sub-optimal filter using simulations and confirmed that it outperforms the existing best linear filter. For example, when the observation noise is a Gaussian noise plus an  $\alpha$ -stable noise with index 1.4, the mean square error of the sub-optimal filter is 3.86 while that of the best linear filter is 180.26.

I described the results in the article *Optimal Filtering of a Gaussian Signal in the presence of Lévy Noise*, and submitted to the SIAM journal of Applied Mathematics for publication. The paper is accepted for publication provided minor changes are incorporated.

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